defined a smoothing spline functional with sampling measure weights. The equivalent number of parameters dened on this functional does not depend on the distri butions of samples. The approximation of the equivalent number of parameters is derived when the number of samples becomes infinity. This approximation greatly reduced the calculation time needed to estimate the opti² **mal smoothing. The smoothing spline calculation cost** was so high that new algorithms (FMMEast multipole method) were introduced and we developed the smooth \boxtimes ing engine which was applied to practical problems. The **engine generated clear surfaces and was robust to vari**

 $\{ d_M \}$ $\{c\}$ $\{Z_M\}$ $\{O_3\}$ 1 $\frac{1}{3\times 3}$ $\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$ *MM M M M T M* A_M γ $_M$ P_M $\{d_M\}$ $\{z\}$ P_M $Q_{3\times 3}$ $\{C\}$ $\{C\}$ γ μ $^{-}$ × $A_{\scriptscriptstyle{M}}$ γ M $P_{\scriptscriptstyle{M}}$ $\{d_{\scriptscriptstyle{M}}\}$ $\{z_{\scriptscriptstyle{M}}\}$

5. Plate Surface Estimation by Using LIDAR and Smoothing TPS with Sampling Measure Weights

Figure 2 shows the point cloud of samples measured of samples measured of samples measured of samples measured with a LIDAR on a LIDAR on a LIDAR on a plate surface. The plate surface is μ 5.475 m and the plate width is 2.143 m. The -axis is the rolling direction, and the values of the values of the values of the α ten times the axis is the axis is the axis is the plate width ℓ_{ℓ}

direction. The distance in the longitudinal direction is decoupled because the wavelength in the longitudinal direction is shorter than that in the width direction and to show both wavelengths in the same dimension. The distribution of samples is not homogeneous. The number of samples is 25 691. The values of the samples include measurement error of 2 mm. This error value is taken from the 3-D laser specification of the 3-D laser scanner, and specification of the 3-D laser scanner, a which is a Photon 120 manufactured by FARO manufactured by \mathcal{F}_A Fig. 2.) and interpolated into a \mathcal{F}

 $\int\limits_{1}^{1} \omega _{M}^{(I)}\left| Z_{M}^{(I)}-f\Big(X_{M}^{(I)},Y_{M}^{(I)}\Big)\right| ^{-2}$ 2 $\omega_M^{(1)}$ $Z_M^{(1)} - f(x_M^{(1)})$ $lm \, m \, 1$ $A = m$ *m* $\omega_M^{(i)}$ $Z_M^{(i)} - f(x_M^{(i)}, y_M^{(i)})$ *A BIC*_A = *m* $\omega_M^{(1)}$ $Z_M^{(1)} - f(x_M^{(1)}, y_M^{(2)})$ m m 1 $\frac{k_A}{m}$ ω $= m$ $m \frac{m}{\omega_M^{(i)}} z_M^{(i)} - f(x_M^{(i)}, y_M^{(i)})^2$ $i=1$ $\cdots M$ \cdots \cdots $\frac{k_A}{4}$

smoothing TPS system with sampling measure $\mathcal{L}_\mathcal{F}$

- weights were solved theoretically. (3) The approximation of ENOP was derived theoreti cally from the frequency response function. We confirmed that the approximation agreed with the theoretical ENOP.
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- ∂ the information criteria including the approximation ∂ tion of ENOP enabled calculation of the optimal of smoothing parameter.
- $\hat{\bm{J}}$ we applied the problem of the p actual large-scale samples measured by LIDAR.
- The results clarified the fact that engineering appli
- cations of the method are possible.

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